ydiv ($\mathbf{t_0}$, *numdiv*, *months*, *anchor*, *endmnth*)

Returns the date of the beginning of the year division (e.g. the beginning of the current quarter) in which given date falls, or that is a given number of divisions away.

Required Parameters	Description	Example
$\mathbf{t_0}$	given date	date(97,6,15)

Optional Parameters	Description	Default	Example
numdiv	number of divisions away from beginning of the division in which $\mathbf{t_0}$ falls (can be negative)	0 (current)	20
months	number of months per division e.g. 3 = quarter, 6 = half-year; does not have to be a divisor of 12, e.g. 5; can be longer than 1 year, e.g. 18	3 (quarter)	6 (half-year)
anchor	date to anchor division boundaries on	date(0,1,1)	date(95,4,15)
endmnth	force adherence to ends of months 1 = on and 0 = off	1 (on)	0 (off)

Description: A division is a number of months; usually an integral multiple of divisions comprises a year. Examples of divisions are a quarter (three months), half-year (six months), and sixth (two months). This function calculates the date of the beginning of the year division in which $\mathbf{t_0}$ falls. For example, the function can be used to calculate the beginning of the quarter in which October 11, 1995 falls.

Moreover, the function can calculate the date which is a given number of divisions away from the beginning of the current division. The optional parameter *numdiv* specifies the offset from the current division. For example, the function can be used to calculate the beginning of the fifth quarter after the quarter containing March 21, 1995. In this case, the *numdiv* parameter should be passed as 5.

By default, divisions are three month long quarters. This length can be changed via the optional parameter *months* which dictates the number of months per division. If, for example, half-years are desired, this optional parameter should be passed as 6. The parameter need not be an integral divisor of 12: For example, divisions of 7 months each are perfectly acceptable.

By default, divisions coincide with January 1, 1900; that is, January 1, 1900 is a division boundary, and all other boundaries are some number of divisions away from it. Thus, the default quarter boundaries are January 1, April 1, July 1 and October 1 of each year. The default third boundaries are January 1, May 1 and September 1 of each year. In the case of divisions that are 5 months long, the boundaries would be January 1, 1900, June 1, 1900, November 1, 1900, April 1, 1901, and so on, in steps of five months.

When the division length *months* is an integral divisor of 12 (1, 2, 3, 4, 6 or 12), division boundaries fall on the same dates each year. Other division lengths, including those exceeding a year, result in more complex cycles such the five-month divisions illustrated above.

Division cycles can be changed to coincide with any date via the optional parameter *anchor*. For example, if the desired quarter boundaries are February 23, May 23, August 23, and November 23, then any one of these dates, in any year, can be passed as *anchor*. When the division length is an integral divisor of 12, the year component of *anchor* is ignored since the boundary dates will repeat every year.

If *anchor* falls on the last day of a month with less than 31 days, the *endmnth* parameter determines if ***ydiv*** will always return the last day of a month (see the discussion in ***amnths*** on page 39).

Example:

Problem: What is the date of the beginning of the quarter in which August 19, 1995 falls (assuming that quarters begin January 1, April 1, July 1 and October 1)?

Solution: ***ydiv***(E22)
where cell E22 contains date(95,8,19).

Answer: July 1, 1995.

Example:

Problem: What is the date of the beginning of the quarter that is six quarters before the quarter containing August 19, 1995 (assuming that quarters begin January 1, April 1, July 1 and October 1)?

Solution: ***ydiv***(E22, −6)
where cell E22 contains date(95,8,19).

Answer: January 1, 1994.

Example:

Problem: A Treasury bond pays a coupon every six months until the bond's maturity, at which time it pays a final coupon. What is the date of the next coupon payment after the date September 12, 1995, if the bond matures July 15, 1999?

Solution: ***ydiv***(B21, 1, 6, A14)
where cell B21 contains date(95,9,12) and cell A14 contains date(99,7,15). The value 1 is passed as the *numdiv* parameter to specify the beginning of the next division (next coupon payment date). The value 6 is passed as the *months* parameter indicating that divisions are six months long. The *anchor* parameter specifies that year divisions must coincide with that date, i.e., coupon dates coincide with the maturity date.

Answer: January 15, 1996.

Restrictions:

- *numdiv* must be an integer, if specified
- *months* must be a positive integer, if specified
- *endmnth* must equal 1 or 0, if specified

byld (**coupon**, **maturity**, **settle**, **price**, *dated*, *firstcoup*, *bondtype*, *freq*, *redem*, *itax*, *gtax*, *cutoff*, *issue_pr*, *true_yld*, *wkend*, *holidays*, [*r_out*], [*r_start*], [*r_end*], [*r_amount*], [*voluntary*], *numexdiv*)

Calculates the yield-to-maturity (or yield-to-call) for a bond given price.

Required Parameters	Description	Example
coupon	coupon rate, expressed as a decimal fraction (or a range of rates for stepped coupon bonds)	0.084
maturity	maturity date (or a range of dates for stepped coupon bonds)	date(105,6,30)
settle	settlement date	date(98,6,16)
price	quoted price (par 100, not including accrued interest)	99

Optional Parameters	Description	Default	Example
dated	dated date (0 if none)	none (or 0)	date(96,6,1)
firstcoup	first coupon date (0 if none)	none (or 0)	date(97,5,31)
bondtype	bond type code (see page 258 for a complete listing of the codes)	0 (Treasury)	1 (Agency)
freq	frequency of coupon payments, expressed as the number of payments per year (for sinking fund bonds the redemption payment frequency may be specified by making this a two-cell range)	0 (inferred from *bondtype*)	4 (quarterly)
redem	redemption value	100	101.5
itax	marginal tax rate for ordinary income	0	31%
gtax	marginal tax rate for capital gains	0	28%
cutoff	holding period term: if *cutoff* $\leq$ 24, then months from **settle**; if *cutoff* $>$ 24, then days from **settle**	12 (months)	90 (days)
issue_pr	issue price (used with Italian bonds)	100	108
true_yld	flag specifying calculation method: 0 = default calculation method (net yield), 1 = true yield, −1 = ISMA, −2 = simple (Japanese), −3 = Braeß-Fangmeyer, −4 = Moosmüller, −5 = consortium, −6 = force act / act discounting	0	1
wkend	flag specifying whether Saturdays and Sundays are to be considered business days— 0: Sat and Sun are not business days 1: Sun is not a business day 2: Sat is not a business day 3: Sat and Sun are business days	0	3
holidays	A range of holidays (for use with true yield and some bonds which trade ex-dividend)	0	A1..A15

Optional Parameters	Description	Default	Example
[*parameters enclosed in brackets*]	These parameters should be used with sinking bonds only. For a description of these parameters, see the section **Optional Parameters for Sinking Fund Bonds** beginning on page 263.		
numexdiv	number of days the bond trades ex-dividend prior to the coupon date. Use negative integers to specify calendar days; positive integers to specify business days	inferred from *bondtype*	3

Description: The function ***byld*** calculates the yield-to-maturity (or yield-to-call) given price per 100 face value. The given price is the quoted price, and does not include accrued interest. The yield-to-maturity is the internal rate of return of a bond's cash flows. Yield-to-call can be calculated by specifying the call date for maturity and the call price for *redem*. The parameters *itax* and *gtax* allow an after-tax yield to be specified. If *itax* and *gtax* are not specified or are set to 0, then ***byld*** behaves like a normal price to yield function.

byld rounds the yield depending on the value passed for the *bondtype* parameter. Rounding is performed in accordance with the table on page 261.

Calculation. The invoice price of a bond equals the present value (discounted value) of all expected coupon and principal payments. The interest rate used for discounting the payments, such that present value equals the invoice price, is the yield-to-maturity (yield-to-call).

While there are closed formulas for calculating bond price given yield-to-maturity, there are none for calculating yield-to-maturity given price. Thus, yield is calculated using Newton-Raphson search, an iterative trial and error algorithm, over formulas for computing price given yield. For a description of these price formulas, see discussion of ***bpr***.

For purposes of computing after-tax yield-to-maturity, income is split into two components: coupon interest (ordinary income) and capital gain income:

$$\mathbf{price} + ai(1 - itax) = \mathbf{coupon} \cdot (1 - itax) \sum_{i=1}^{n} \left(1 + \frac{y_t}{freq}\right)^{-\tau_i} + [redem - (redem - \mathbf{price}) \cdot gtax] \left(1 + \frac{y_t}{freq}\right)^{-\tau_n},$$

where τ_i is the time from settlement to the i^{th} payment, y_t is the yield after taxes, n is the number of coupon payments, and **price** is the quoted price, which does not include any accrued interest.

The optional parameter *cutoff* specifies how long after the settlement date (**settle**) some of the bond income can be treated as a long-term capital gain. The parameters **settle** and *cutoff* imply a cutoff date. The cutoff date is the settlement date plus the number of months or days specified by cutoff. If the bond matures after the cutoff date, then it is eligible for long-term capital gains tax treatment. For example, if *cutoff* is passed the value 6 then the cutoff date is six months after **settle**, but if *cutoff* is passed the value 45 then it is 45 days after **settle**.

Other tax issues arise when using Italian bonds (*bondtype* = 260–263). Here one needs to specify the issue price *issue_pr* in order to account for the withholding taxes charged on a portion of the difference between the redemption value and the issue price. The default value for *issue_pr* is 100.

Ignoring Odd First Coupons for Municipal Bonds. Industry standard price and yield quotes for municipal bonds do not treat an odd first coupon period specially. If industry standard quotes are desired for a municipal bond, one should use the *bondtype* codes 16 and 17. If more accurate quotes, which take the odd coupons into account, are desired, use the *bondtype* codes 2 and 12. (For bonds which have no odd coupons, the different codes give the same results.)

Example:

Problem: *Zero Coupon Treasury Note.* What is the yield-to-maturity of a zero coupon Treasury bond maturing August 17, 2025 for settlement on September 22, 1997, if its price is 9.18?

Solution: ***byld***(0.0, E1, E2, 9.18)
where cell E1 contains date(125,8,17) and cell E2 contains date(97,9,22).

Answer: The yield-to-maturity is 8.744767%.

Example:

Problem: *Callable Treasury Bond.* Consider an 11.75% Treasury bond maturing on November 15, 2022, and callable at par five years before maturity. If the bond is priced at 126.1875 for settlement on January 23, 1997, what is the yield-to-call? What is the yield-to-maturity?

Solution: The call date is November 15, 2017, and the call price (redemption value) is 100. Thus, the yield-to-call can be calculated using:
byld(0.1175, D1, D2, 126.1875)
where cell D1 contains date(117,11,15) and cell D2 contains date(97,1,23).

The yield-to-maturity can be calculated using:
byld(0.1175, D3, D2, 126.1875)
where cell D3 contains date(122,11,15).

Answer: The yield-to-call is 8.950722%; the yield-to-maturity 9.097807%.

Example:

Problem: *Corporate Bond in Odd First Coupon Period.* Consider an 11.75% corporate bond maturing on May 31, 2006, issued on March 1, 1997, and paying its first coupon on November 30, 1997. If the bond is priced at 99.8125 for settlement on March 23, 1997, what is the yield-to-maturity?

Solution: The yield-to-maturity can be calculated using:
byld(11.75%, A25, A26, 99.8125, A27, A28, 13)
where cell A25 contains date(2006,5,31), cell A26 contains date(97,3,23), cell A27 contains date(97,3,1) and cell A28 contains date(97,11,30). A value of 13 is passed for *bondtype* to specify an end-of-month corporate bond.

Answer: The yield-to-maturity is 11.760%.

Example:

Problem: *Italian Bond.* What is the yield for a 10-year 9.5% Italian BTP government bond maturing January 11, 2005 and settling May 23, 1997? The current price is 95.75 and the issue price is 95.50.

Solution: ***byld***(0.095, A1, A2, 95.75, A3, 0, 260, 0, 100, 0.0, 0.125, 12, 95.50)
where cell A1 contains date(105,1,11) and cell A2 contains date(97,5,23). Cell A3 contains date(95,1,11) which is passed for the *dated* parameter indicating the 10 year original maturity of the bond (which is needed to properly account for the withholding tax on the redemption value). Since the settlement date is in a normal coupon period, the *firstcoup* parameter is passed 0, its default value. The parameter *bondtype* is passed the value 260 to indicate an Italian BTP government bond. The parameter *freq* is passed the value 0 so that the frequency of coupon payments is inferred from the *bondtype* code. The redemption value, *redem*, is 100. The parameter *itax* is passed the value 0 but *gtax* is specified as 0.125, which is the required withholding tax on the pre-1997 portion of the OID discount on Italian government bonds. The cutoff parameter is passed its default, 12 (this is ignored for since Italian bonds are treated specially), so that *issue_pr* can be given as 95.50.

Answer: The yield is 10.564%.

Example:

Problem: *Sinking Fund Bonds—Custom Redemption Schedule.* A twenty year corporate sinking fund bond pays 6.25% and matures on March 1, 2014. On February 15, 2010, the bond is trading at 96.45412. The customized redemption schedule for a $50,000 issue is given as:

	B	C	D	E
4	Begin	End	Amount	Fraction
5	March 1, 1995	March 1, 1998	$1500	0.030
6	March 1, 1999	March 1, 2013	$2750	0.055

As of the trade date, $13,750 of the debt remains outstanding. What is the bond's yield?

Solution: ***byld***(0.0625, date(114,3,1), date(110,2,15), 96.45412, 0, 0, 3, A4..A5, 100, 0, 0, 0, 100, 0, 0, 0, 13750 / 50000, B5..B6, C5..C6, E5..E6)
where cells A4 and A5 contain the coupon / redemption frequencies, 2 and 1; cells B5, B6, C5, and C6 contain the begin and end dates respectively and E5 and E6 contain the

amount of payments expressed as fractions of the amount issued (E5 = 1500 / 50000 = 0.030 and E6 = 2750 / 50000 = 0.055).

Answer: The yield-to-maturity is 8.240%.

See the section **Optional Parameters for Sinking Fund Bonds** beginning on page 263 for details on entering parameters for sinking fund bonds.

Example:

Problem: *Sinking Fund Bonds—Uniform Redemption Schedule.* The Dutch government ten year 10.5% Nederland bond matures on May 1, 2004. On September 5, 1998 the bond is trading at a clean price of 115. The bond pays an annual coupon and has a uniform redemption schedule beginning in 2000. What is the yield for the bond?

Solution: ***byld***(0.105, date(104,5,1), date(98,9,5), 115, 0, 0, 160, 1, 100, 0, 0, 0, 100, 0, 0, 0, −1, date(100,5,1))

The parameter *bondtype* is passed the value 160 to indicate a Dutch bond. −1 is passed for the parameter *r_out* to indicate a uniform redemption schedule. date(100,5,1) is passed as the date on which sinking fund payments begin, *r_start*.

Answer: The yield to maturity is 5.7%.

See the section **Optional Parameters for Sinking Fund Bonds** beginning on page 263 for details on entering parameters for sinking fund bonds.

Example:

Problem: *Stepped Coupon Bonds.* Consider a stepped coupon bond with the following conversion rate schedule:

	A	B	C
1			coupon
2	settlement date:	1 / 15 / 1998	0.0455
3	conversion dates:	7 / 27 / 1998	0.050
4		7 / 27 / 2000	0.0620
5		7 / 27 / 2002	0.0660
6	maturity date:	7 / 27 / 2004	

If the bond is trading at 100.25, what is the yield-to-maturity?

Solution: ***byld***(C2..C5, B3..B6, B2, 100.25)

where C2..C5 is a range of cells containing the coupon rates and B2..B6 is a range of cells containing the settlement date and conversion dates, with the last date being the maturity date.

Answer: The yield-to-maturity is 5.69%.

See the section **Required Parameters for Stepped Coupons** on page 255 for details on entering parameters for stepped coupon bonds.

Restrictions:

- If **price** is such that it does not allow convergence in the Newton-Raphson search, an error value is returned.

Also see the section **Parameter Restrictions** beginning on page 267.